

# Statistics 350 Help Card

## Summary Measures

### Sample Mean

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum x_i}{n}$$

### Sample Standard Deviation

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$$

## Probability Rules

- **Complement rule**

$$P(A^C) = 1 - P(A)$$

- **Addition rule**

General:  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

For independent events:

$$P(A \text{ or } B) = P(A) + P(B) - P(A)P(B)$$

For mutually exclusive events:  $P(A \text{ or } B) = P(A) + P(B)$

- **Multiplication rule**

General:  $P(A \text{ and } B) = P(A)P(B | A)$

For independent events:  $P(A \text{ and } B) = P(A)P(B)$

For mutually exclusive events:  $P(A \text{ and } B) = 0$

- **Conditional Probability**

General:  $P(A | B) = \frac{P(A \text{ and } B)}{P(B)}$

For independent events:  $P(A | B) = P(A)$

For mutually exclusive events:  $P(A | B) = 0$

## Discrete Random Variables

### Mean

$$E(X) = \mu = \sum x_i p_i = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k$$

### Standard Deviation

$$s.d.(X) = \sigma = \sqrt{\sum (x_i - \mu)^2 p_i} = \sqrt{\sum (x_i^2 p_i) - \mu^2}$$

## Binomial Random Variables

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\text{where } \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

### Mean

$$E(X) = \mu_X = np$$

### Standard Deviation

$$s.d.(X) = \sigma_X = \sqrt{np(1-p)}$$

## Normal Random Variables

- $z\text{-score} = \frac{\text{observation} - \text{mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$

- Percentile:  $x = z\sigma + \mu$

- If  $X$  has the  $N(\mu, \sigma)$  distribution, then the variable

$$Z = \frac{X - \mu}{\sigma} \text{ has the } N(0,1) \text{ distribution.}$$

## Normal Approximation to the Binomial Distribution

If  $X$  has the  $B(n, p)$  distribution and the sample size  $n$  is large enough (namely  $np \geq 10$  and  $n(1-p) \geq 10$ ),

then  $X$  is approximately  $N(np, \sqrt{np(1-p)})$ .

## Sample Proportions

$$\hat{p} = \frac{x}{n}$$

### Mean

$$E(\hat{p}) = \mu_{\hat{p}} = p$$

### Standard Deviation

$$s.d.(\hat{p}) = \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

### Sampling Distribution of $\hat{p}$

If the sample size  $n$  is large enough (namely,  $np \geq 10$  and  $n(1-p) \geq 10$ )

then  $\hat{p}$  is approximately  $N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$ .

## Sample Means

### Mean

$$E(\bar{X}) = \mu_{\bar{X}} = \mu$$

### Standard Deviation

$$s.d.(\bar{X}) = \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

### Sampling Distribution of $\bar{X}$

If  $X$  has the  $N(\mu, \sigma)$  distribution, then  $\bar{X}$  is

$$N(\mu_{\bar{X}}, \sigma_{\bar{X}}) \Leftrightarrow N\left(\mu, \frac{\sigma}{\sqrt{n}}\right).$$

If  $X$  follows *any* distribution with mean  $\mu$  and standard deviation  $\sigma$  and  $n$  is large,

then  $\bar{X}$  is approximately  $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ .

This last result is **Central Limit Theorem**

Population Proportion	Two Population Proportions	Population Mean
<b>Parameter</b> $p$	<b>Parameter</b> $p_1 - p_2$	<b>Parameter</b> $\mu$
<b>Statistic</b> $\hat{p}$	<b>Statistic</b> $\hat{p}_1 - \hat{p}_2$	<b>Statistic</b> $\bar{x}$
<b>Standard Error</b> $\text{s.e.}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	<b>Standard Error</b> $\text{s.e.}(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	<b>Standard Error</b> $\text{s.e.}(\bar{x}) = \frac{s}{\sqrt{n}}$
<b>Confidence Interval</b> $\hat{p} \pm z^* \text{s.e.}(\hat{p})$ <b>Conservative Confidence Interval</b> $\hat{p} \pm \frac{z^*}{2\sqrt{n}}$	<b>Confidence Interval</b> $(\hat{p}_1 - \hat{p}_2) \pm z^* \text{s.e.}(\hat{p}_1 - \hat{p}_2)$	<b>Confidence Interval</b> $\bar{x} \pm t^* \text{s.e.}(\bar{x}) \quad \text{df} = n - 1$  <b>Paired Confidence Interval</b> $\bar{d} \pm t^* \text{s.e.}(\bar{d}) \quad \text{df} = n - 1$
<b>Large-Sample z-Test</b> $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	<b>Large-Sample z-Test</b> $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$	<b>One-Sample t-Test</b> $t = \frac{\bar{x} - \mu_0}{\text{s.e.}(\bar{x})} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \quad \text{df} = n - 1$  <b>Paired t-Test</b> $t = \frac{\bar{d} - 0}{\text{s.e.}(\bar{d})} = \frac{\bar{d}}{s_d/\sqrt{n}} \quad \text{df} = n - 1$
<b>Sample Size</b> $n = \left(\frac{z^*}{2m}\right)^2$		

Two Population Means	
General	Pooled
<b>Parameter</b> $\mu_1 - \mu_2$	<b>Parameter</b> $\mu_1 - \mu_2$
<b>Statistic</b> $\bar{x}_1 - \bar{x}_2$	<b>Statistic</b> $\bar{x}_1 - \bar{x}_2$
<b>Standard Error</b> $\text{s.e.}(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	<b>Standard Error</b> $\text{pooled s.e.}(\bar{x}_1 - \bar{x}_2) = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ where $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}}$
<b>Confidence Interval</b> $(\bar{x}_1 - \bar{x}_2) \pm t^* (\text{s.e.}(\bar{x}_1 - \bar{x}_2)) \quad \text{df} = \min(n_1 - 1, n_2 - 1)$	<b>Confidence Interval</b> $(\bar{x}_1 - \bar{x}_2) \pm t^* (\text{pooled s.e.}(\bar{x}_1 - \bar{x}_2)) \quad \text{df} = n_1 + n_2 - 2$
<b>Two-Sample t-Test</b> $t = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\text{s.e.}(\bar{x}_1 - \bar{x}_2)} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \text{df} = \min(n_1 - 1, n_2 - 1)$	<b>Pooled Two-Sample t-Test</b> $t = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\text{pooled s.e.}(\bar{x}_1 - \bar{x}_2)} = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad \text{df} = n_1 + n_2 - 2$

One-Way ANOVA							
SS Groups = $SSG = \sum_{\text{groups}} n_i (\bar{x}_i - \bar{x})^2$	MS Groups = $MSG = \frac{SSG}{k-1}$	ANOVA Table					
SS Error = $SSE = \sum_{\text{groups}} (n_i - 1) s_i^2$	MS Error = $MSE = s_p^2 = \frac{SSE}{N-k}$						
SS Total = $SSTO = \sum_{\text{values}} (x_{ij} - \bar{x})^2$	$F = \frac{\text{MS Groups}}{\text{MS Error}}$	Source	SS	DF	MS	F	
Confidence Interval	$\bar{x}_i \pm t^* \frac{s_p}{\sqrt{n_i}}$	df = $N - k$	Groups	SS Groups	$k - 1$	MS Groups	$F$
			Error	SS Error	$N - k$	MS Error	
		Total	SSTO	$N - 1$			
			Under $H_0$ , the $F$ statistic follows an $F(k - 1, N - k)$ distribution.				

Regression	
<b>Linear Regression Model</b>  <b>Population Version:</b> Mean: $\mu_Y(x) = E(Y) = \beta_0 + \beta_1 x$ Individual: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ where $\varepsilon_i$ is $N(0, \sigma)$  <b>Sample Version:</b> Mean: $\hat{y} = b_0 + b_1 x$ Individual: $y_i = b_0 + b_1 x_i + e_i$	<b>Standard Error of the Sample Slope</b> $s.e.(b_1) = \frac{s}{\sqrt{S_{XX}}} = \frac{s}{\sqrt{\sum (x - \bar{x})^2}}$  <b>Confidence Interval for <math>\beta_1</math></b> $b_1 \pm t^* s.e.(b_1) \quad df = n - 2$  <b>t-Test for <math>\beta_1</math></b> To test $H_0 : \beta_1 = 0$ $t = \frac{b_1 - 0}{s.e.(b_1)} \quad df = n - 2$ or $F = \frac{MSREG}{MSE} \quad df = 1, n - 2$
<b>Parameter Estimators</b> $b_1 = \frac{S_{XY}}{S_{XX}} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = \frac{\sum (x - \bar{x})y}{\sum (x - \bar{x})^2}$ $b_0 = \bar{y} - b_1 \bar{x}$	<b>Confidence Interval for the Mean Response</b> $\hat{y} \pm t^* s.e.(fit) \quad df = n - 2$ where $s.e.(fit) = s \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{S_{XX}}}$
<b>Residuals</b> $e = y - \hat{y} = \text{observed } y - \text{predicted } y$	<b>Prediction Interval for an Individual Response</b> $\hat{y} \pm t^* s.e.(pred) \quad df = n - 2$ where $s.e.(pred) = \sqrt{s^2 + (s.e.(fit))^2}$
<b>Correlation and its square</b> $r = \frac{S_{XY}}{\sqrt{S_{XX} S_{YY}}}$ $r^2 = \frac{SSTO - SSE}{SSTO} = \frac{SSREG}{SSTO}$ where $SSTO = S_{YY} = \sum (y - \bar{y})^2$	<b>Standard Error of the Sample Intercept</b> $s.e.(b_0) = s \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{XX}}}$  <b>Confidence Interval for <math>\beta_0</math></b> $b_0 \pm t^* s.e.(b_0) \quad df = n - 2$  <b>t-Test for <math>\beta_0</math></b> To test $H_0 : \beta_0 = 0$ $t = \frac{b_0 - 0}{s.e.(b_0)} \quad df = n - 2$
<b>Estimate of <math>\sigma</math></b> $s = \sqrt{MSE} = \sqrt{\frac{SSE}{n - 2}} \quad \text{where } SSE = \sum (y - \hat{y})^2 = \sum e^2$	

Chi-Square Tests	
<b>Test of Independence &amp; Test of Homogeneity</b>	<b>Test for Goodness of Fit</b>
<b>Expected Count</b> $E = \text{expected} = \frac{\text{row total} \times \text{column total}}{\text{total } n}$	<b>Expected Count</b> $E_i = \text{expected} = np_{i0}$
<b>Test Statistic</b> $X^2 = \sum \frac{(O - E)^2}{E} = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$ $df = (r - 1)(c - 1)$	<b>Test Statistic</b> $X^2 = \sum \frac{(O - E)^2}{E} = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$ $df = k - 1$
If $Y$ follows a $\chi^2(df)$ distribution, then $E(Y) = df$ and $\text{Var}(Y) = 2(df)$ .	

Table entry for z is the area to the left of z

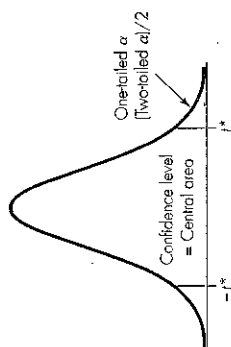
TABLE A.1 ■ Standard Normal Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

In the Extreme

z	6.00	5.61	5.20	4.75	4.26	3.72	3.09	-6.00	-5.61	-5.20	-4.75	-4.26	-3.72	-3.09
Probability	.999999999	.999999999	.999999999	.999999999	.999999999	.999999999	.999999999	.000000001	.000000001	.000000001	.000000001	.000000001	.000000001	.001

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Table A.2  $t^*$  Multipliers for Confidence Intervals and Rejection Region Critical Values

df	Confidence Level						
	.80	.90	.95	.98	.99	.998	.999
1	3.08	6.31	12.71	31.82	63.66	318.31	636.62
2	1.89	2.92	4.30	6.96	9.92	72.33	31.60
3	1.64	2.35	3.18	4.54	5.84	10.21	12.92
4	1.53	2.13	2.78	3.75	4.60	7.17	8.61
5	1.48	2.02	2.57	3.36	4.03	5.89	6.87
6	1.44	1.94	2.45	3.14	3.71	5.21	5.96
7	1.41	1.89	2.36	3.00	3.50	4.79	5.41
8	1.40	1.86	2.31	2.90	3.36	4.50	5.04
9	1.38	1.83	2.26	2.82	3.25	4.30	4.78
10	1.37	1.81	2.23	2.76	3.17	4.14	4.59
11	1.36	1.80	2.20	2.72	3.11	4.02	4.44
12	1.36	1.78	2.18	2.68	3.05	3.93	4.32
13	1.35	1.77	2.16	2.65	3.01	3.85	4.22
14	1.35	1.76	2.14	2.62	2.98	3.79	4.14
15	1.34	1.75	2.13	2.60	2.95	3.73	4.07
16	1.34	1.75	2.12	2.58	2.92	3.69	4.01
17	1.33	1.74	2.11	2.57	2.90	3.65	3.97
18	1.33	1.73	2.10	2.55	2.88	3.61	3.92
19	1.33	1.73	2.09	2.54	2.86	3.58	3.88
20	1.33	1.72	2.09	2.53	2.85	3.55	3.85
21	1.32	1.72	2.08	2.52	2.83	3.53	3.82
22	1.32	1.72	2.07	2.51	2.82	3.50	3.79
23	1.32	1.71	2.07	2.50	2.81	3.48	3.77
24	1.32	1.71	2.06	2.49	2.80	3.47	3.75
25	1.32	1.71	2.06	2.49	2.79	3.45	3.73
26	1.31	1.71	2.06	2.48	2.78	3.43	3.71
27	1.31	1.70	2.05	2.47	2.77	3.42	3.69
28	1.31	1.70	2.05	2.47	2.76	3.41	3.67
29	1.31	1.70	2.05	2.46	2.76	3.40	3.66
30	1.31	1.70	2.04	2.46	2.75	3.39	3.65
40	1.30	1.68	2.02	2.42	2.70	3.31	3.55
50	1.30	1.68	2.01	2.40	2.68	3.26	3.50
60	1.30	1.67	2.00	2.39	2.66	3.23	3.46
70	1.29	1.67	1.99	2.38	2.65	3.21	3.44
80	1.29	1.66	1.99	2.37	2.64	3.20	3.42
90	1.29	1.66	1.99	2.37	2.63	3.18	3.40
100	1.29	1.66	1.98	2.36	2.63	3.17	3.39
1000	1.282	1.646	1.962	2.330	2.581	3.098	3.300
Infinite	1.281	1.645	1.960	2.326	2.576	3.090	3.291
Two-tailed $\alpha$	.20	.10	.05	.02	.01	.002	.001
One-tailed $\alpha$	.10	.05	.025	.01	.005	.001	.0005

TABLE A.3 ■ One-Sided  $p$ -Values for Significance Tests Based on a  $t$ -Statistic

- ☛ A  $p$ -value in the table is the area to the right of  $t$ .
- ☛ Double the value if the alternative hypothesis is two-sided (not equal).

Absolute Value of t-Statistic										
df	1.28	1.50	1.65	1.80	2.00	2.33	2.58	3.00		
1	0.211	0.187	0.173	0.161	0.148	0.129	0.118	0.102		
2	0.164	0.136	0.120	0.107	0.092	0.073	0.062	0.048		
3	0.145	0.115	0.099	0.085	0.070	0.051	0.041	0.029		
4	0.135	0.104	0.087	0.073	0.058	0.040	0.031	0.020		
5	0.128	0.097	0.080	0.066	0.051	0.034	0.025	0.015		
6	0.124	0.092	0.075	0.061	0.046	0.029	0.021	0.012		
7	0.121	0.089	0.071	0.057	0.043	0.026	0.018	0.010		
8	0.118	0.086	0.069	0.055	0.040	0.024	0.016	0.009		
9	0.116	0.084	0.067	0.053	0.038	0.022	0.015	0.007		
10	0.115	0.082	0.065	0.051	0.037	0.021	0.014	0.007		
11	0.113	0.081	0.064	0.050	0.035	0.020	0.013	0.006		
12	0.112	0.080	0.062	0.049	0.034	0.019	0.012	0.006		
13	0.111	0.079	0.061	0.048	0.033	0.018	0.011	0.005		
14	0.111	0.078	0.061	0.047	0.033	0.018	0.011	0.005		
15	0.110	0.077	0.060	0.046	0.032	0.017	0.010	0.004		
16	0.109	0.077	0.059	0.045	0.031	0.017	0.010	0.004		
17	0.109	0.076	0.059	0.045	0.031	0.016	0.010	0.004		
18	0.108	0.075	0.058	0.044	0.030	0.016	0.009	0.004		
19	0.108	0.075	0.058	0.044	0.030	0.015	0.009	0.004		
20	0.108	0.075	0.057	0.043	0.030	0.015	0.009	0.004		
21	0.107	0.074	0.057	0.043	0.029	0.015	0.009	0.003		
22	0.107	0.074	0.057	0.043	0.029	0.015	0.009	0.003		
23	0.107	0.074	0.056	0.042	0.029	0.014	0.008	0.003		
24	0.106	0.073	0.056	0.042	0.028	0.014	0.008	0.003		
25	0.106	0.073	0.056	0.042	0.028	0.014	0.008	0.003		
26	0.106	0.073	0.055	0.042	0.028	0.014	0.008	0.003		
27	0.106	0.073	0.055	0.042	0.028	0.014	0.008	0.003		
28	0.106	0.072	0.055	0.041	0.028	0.014	0.008	0.003		
29	0.105	0.072	0.055	0.041	0.027	0.013	0.008	0.003		
30	0.105	0.072	0.055	0.041	0.027	0.013	0.008	0.003		
40	0.104	0.071	0.053	0.040	0.026	0.012	0.007	0.002		
50	0.103	0.070	0.053	0.039	0.025	0.012	0.006	0.002		
60	0.103	0.069	0.052	0.038	0.025	0.012	0.006	0.002		
70	0.102	0.069	0.052	0.038	0.025	0.011	0.006	0.002		
80	0.102	0.069	0.051	0.038	0.024	0.011	0.006	0.002		
90	0.102	0.069	0.051	0.038	0.024	0.011	0.006	0.002		
100	0.102	0.068	0.051	0.037	0.024	0.011	0.006	0.002		
1000	0.100	0.067	0.050	0.036	0.023	0.010	0.005	0.001		
Infinite	0.1003	0.0658	0.0495	0.0359	0.0228	0.0099	0.0049	0.0013		

Note that the  $t$ -distribution with infinite df is the standard normal distribution.

**TABLE A.5 ■ Chi-square Distribution**

df	<i>p</i> = Area to Right of Chi-square Value								
	0.50	0.25	0.10	0.075	0.05	0.025	0.01	0.005	0.001
1	0.45	1.32	2.71	3.17	3.84	5.02	6.63	7.88	10.83
2	1.39	2.77	4.61	5.18	5.99	7.38	9.21	10.60	13.82
3	2.37	4.11	6.25	6.90	7.81	9.35	11.34	12.84	16.27
4	3.36	5.39	7.78	8.50	9.49	11.14	13.28	14.86	18.47
5	4.35	6.63	9.24	10.01	11.07	12.83	15.09	16.75	20.51
6	5.35	7.84	10.64	11.47	12.59	14.45	16.81	18.55	22.46
7	6.35	9.04	12.02	12.88	14.07	16.01	18.48	20.28	24.32
8	7.34	10.22	13.36	14.27	15.51	17.53	20.09	21.95	26.12
9	8.34	11.39	14.68	15.63	16.92	19.02	21.67	23.59	27.88
10	9.34	12.55	15.99	16.97	18.31	20.48	23.21	25.19	29.59
11	10.34	13.70	17.28	18.29	19.68	21.92	24.73	26.76	31.26
12	11.34	14.85	18.55	19.60	21.03	23.34	26.22	28.30	32.91
13	12.34	15.98	19.81	20.90	22.36	24.74	27.69	29.82	34.53
14	13.34	17.12	21.06	22.18	23.68	26.12	29.14	31.32	36.12
15	14.34	18.25	22.31	23.45	25.00	27.49	30.58	32.80	37.70
16	15.34	19.37	23.54	24.72	26.30	28.85	32.00	34.27	39.25
17	16.34	20.49	24.77	25.97	27.59	30.19	33.41	35.72	40.79
18	17.34	21.60	25.99	27.22	28.87	31.53	34.81	37.16	42.31
19	18.34	22.72	27.20	28.46	30.14	32.85	36.19	38.58	43.82
20	19.34	23.83	28.41	29.69	31.41	34.17	37.57	40.00	45.31
21	20.34	24.93	29.62	30.92	32.67	35.48	38.93	41.40	46.80
22	21.34	26.04	30.81	32.14	33.92	36.78	40.29	42.80	48.27
23	22.34	27.14	32.01	33.36	35.17	38.08	41.64	44.18	49.73
24	23.34	28.24	33.20	34.57	36.42	39.36	42.98	45.56	51.18
25	24.34	29.34	34.38	35.78	37.65	40.65	44.31	46.93	52.62
26	25.34	30.43	35.56	36.98	38.89	41.92	45.64	48.29	54.05
27	26.34	31.53	36.74	38.18	40.11	43.19	46.96	49.65	55.48
28	27.34	32.62	37.92	39.38	41.34	44.46	48.28	50.99	56.89
29	28.34	33.71	39.09	40.57	42.56	45.72	49.59	52.34	58.30
30	29.34	34.80	40.26	41.76	43.77	46.98	50.89	53.67	59.70



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