Series Tests

Test Name	Series	Convergence Conditions	Divergence Conditions	Comments
Alternating Series	$\sum_{n=1}^{\infty}(-1)^{n-1}a_n$	$0 < a_{n+1} < a_n$ $\lim_{n\to\infty} a_n = 0$		Remainder: $\lvert R_N\rvert < \lvert a_{N+1}\rvert$
Direct Comparison $(a_n, b_n > 0)$	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \le b_n$ $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \le a_n$ $\sum_{n=1}^{\infty} b_n$ diverges	
Geometric Series	$\sum_{n=1}^{\infty} ar^{n-1}$	$-1 < r < 1$	$r \le -1$ or $r \ge 1$	$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$
Integral **(f continuous, positive, decreasing)**	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n) \ge 0$	$\int_1^{\infty} f(x)dx$ converges	$\int_1^{\infty} f(x)dx$ diverges	Remainder: $\int_{N+1}^{\infty} f(x)dx < R_N$ $R_N < \int_N^{\infty} f(x)dx$
Limit Comparison $(a_n, b_n > 0)$	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Test for Divergence	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	Cannot be used to show convergence; Converse is false
p **series**	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$0 < p \le 1$	
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert < 1$	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert > 1$	If $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = 1$ test is inconclusive
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$	If $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$ test is inconclusive
Telescoping	$\sum_{n=1}^{\infty}(b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		$\sum_{n=1}^{\infty}(b_n - b_{n+1}) = b_1 - L$