The 2023 Summer Conference on Topology and Its Applications

Hosted by

Youngstown State University

Supported by

The National Science Foundation

The College of STEM, Youngstown State University

Invited Speakers

Ana Anusic
University of Sao Paulo

Dana Bartošová
University of Florida

William Brian
The University of North Carolina at Charlotte

Maria M. Clementino
Universidade de Coimbra

Jason DeBlois
University of Pittsburgh

Xabier Dominguez
Universidade da Coruña

Goran Erceg
University of Split

Jan Grebik
University of Warwick

Dirk Hofmann
Universidade de Aveiro

Xiaodong Jia
Hunan University

Jimmie Lawson
Louisiana State University

Christopher Leininger
Rice University

Sergio Macias
Universidad Nacional Autónoma de México

Mária Minárová
Slovak University of Technology Bratislava

Yinhe Peng
Chinese Academy of Sciences, Beijing

Candice Price
Smith College

Vinod P. Saxena
Jiwaji University

Isar Stubbe
Université du Littoral-Côte d'Opale

Dexue Zhang
Sichuan University

Session Organizers

1. Topology in Data Science for Natural and Social Sciences

Janet Best (The Ohio State University)
Milan Stehlik (Universidad de Valparaíso, Chile)

2. Category Theory and Computational Topology

David Holgate (University of the Western Cape)
Sanjeevi Krishnan (The Ohio State University)
Josef Šlapal (Brno University of Technology)

3. Algebra and Geometry

Jim Fowler (The Ohio State University)
Salvador Hernandez (Universitat Jaume I)
Gabor Lukacs
Benjamin Linowitz (Oberlin College)

4. General and Set-Theoretic Topology

Ziqin Feng (Auburn University)
Paul Gartside (University of Pittsburgh)
Paul Szeptycki (York University)

5. Dynamics and Continuum Theory

Logan Hoehn (Nipissing University)
James Kelly (Christopher Newport University)
Jonathan Meddaugh (Baylor University)

6. Asymmetry and Many-Valuedness

Tomasz Kubiak (Adam Mickiewicz University, Poznań)
Jimmie Lawson (Louisiana State University)
Frederic Mynard (New Jersey City University)
Olivier O. Otafudu (North-West University)

7. Enriched Categories and Topology

Dirk Hofmann (Universidade de Aveiro)
Ulrich Höhle (Bergische Universität Wuppertal)
Isar Stubbe (Université du Littoral-Côte d'Opale)
Walter Tholen (York University)

Conference Organizers

Benjamin Linowitz (Oberlin College)

Stephen E. Rodabaugh (Youngstown State University)

Jamal K. Tartir (Youngstown State University)

We also recognize conference organizer Austin Melton and session organizers Lori Alvin, Michael Harrison, Ralph Kopperman (1942– 2021), Candice Price, and David Schmidt for their service in 2020.

Conference Schedule

Monday

9:00 – 9:40	Check-in (WCBA)*
9:40 – 10:00	Opening remarks (2212)
10:00 – 10:50	Jimmie Lawson (2212)
11:00 – 11:50	Dana Bartošová (1112) Maria Manuel Clementino (2212)
12:00 – 1:30	Lunch
1:30 – 2:20	Candice Price (2212) (V)
2:30 – 2:45	Coffee break
2:45 – 4:00	Contributed talks
4:10 – 5:00	Goran Erceg (1112) Dirk Hofmann (2212)

Tuesday

9:00 – 9:50	Ana Anusic (2212)
10:00 – 10:15	Coffee break
10:15 – 11:30	Contributed talks
11:40 – 12:30	Sergio Macias (V) (1112) Mária Minárová (2212) (V)
12:30 – 2:00	Lunch
2:00 – 2:50	Christopher Leininger (2212)
3:00 – 3:15	Coffee break
3:15 – 4:30	Contributed talks
4:40 – 5:30	Xabier Dominguez (1112) Isar Stubbe (2212)

Wednesday

9:00 – 9:50	Jan Grebik (2212)
10:00 – 5:30	Excursion

Thursday

9:00 – 9:50	Xiaodong Jia (2212) (V)
10:00 – 10:15	Coffee break
10:15 – 11:30	Contributed talks
11:40 – 12:30	Jason DeBlois (1112) Dexue Zhang (2212) (V)
12:30 – 2:00	Lunch
2:00 – 2:50	William Brian (2212)
3:00 – 3:15	Coffee break
3:15 – 4:50	Contributed talks
6:30 – 8:30	Conference Dinner (Kilcawley Center)

Friday

9:00 – 9:50	Yinhe Peng (1112) (V) Vinod P. Saxena (2212) (V)
10:00 – 10:15	Coffee break
10:15 – 11:45	Contributed talks
11:55 – 12:10	Closing remarks
2:00 – 4:00	Butler Museum of American Art

* All talks are in the Williamson College of Business and Finance.

Contributed Talks

Monday

	Room 1112	Room 2212	Room 2221
2:45 – 3:05	Atish J. Mitra (V)	Daniel Ingebretson	David Holgate
3:10 – 3:30	Yunita S. Anwar (V)	Andrea Ammerlaan	T M G Ahsanullah
3:35 – 3:55	Ziqin Feng		Walter Tholen

Tuesday

	Room 1112	Room 2212	Room 2221
10:15 – 10:35	Daniel S. Farley	Adam Bartoš (V)	Rory Lucyshyn-Wright (V)
10:40 – 11:00	Othman E. Echi (V)	Joseph M. Bedich	Jason Parker (V)
11:05 – 11:25	Ivan Jelić (V)	Lori Alvin	Steven Clontz (V)
3:15 – 3:35	Khulod Almontashery	Mathew T. Timm	Nick Gurski
3:40 – 4:00	Piotr Szewczak	Alejandro Illanes	Stephen E. Rodabaugh
4:05 – 4:25	Akira Iwasa (V)	Iztok Banic	Tom Richmond

Thursday

	Room 1112	Room 2212	Room 2221
10:15 – 10:35	Artur Piekosz (V)	Teja Kac	
10:40 – 11:00	John H. Johnson Jr. (V)	Judy Kennedy	
11:05 – 11:25	Daniel A. Ramras	Daniel A. Ramras	
3:15 – 3:35	Alexandre Karassev	Michelle C. LeMasurier	
3:40 – 4:00	Ignat Soroko	Ulises Morales-Fuentes (V)	
4:05 – 4:25	Peter J. Nyikos (V)	Van Nall	
4:30 – 4:50		Abdul G. Khan (V)	

Friday

	Room 1112	Room 2212	Room 2221
10:15 – 10:35	Lee Kennard	Piotr Oprocha (V)	
10:40 – 11:00	Eliza Wajch (V)	Rene G. Rogina	
11:05 – 11:25	Frederic Mynard (V)	Sina Greenwood	
11:30 – 11:50	Francis G. Wagner (V)	Benjamin Vejnar (V)	

Abstracts

T M G Ahsanullah
King Saud University

Categorical aspects of group objects in some monoidal closed categories

Motivated by the work of Zhang-Fang-Wang, [4] on monoidal closed category ($\mathsf{S}\mathsf{V}$-**GCov**,$\otimes$), where $\otimes$ is a tensor product between two stratified V-generalized convergence spaces, and V stands for complete residuated lattice, we revisit our previous work [1] on enriched lattice-valued stratified convergence groups in an attempt to generalize these structures in the perspective of monoidal closed category; here at this stage, we concentrate only two monoidal closed categories. Thus, exploring monoidal closedness of quantale-valued probabilistic convergence spaces under triangular norm due to Herrlich-Zhang [2], we look at group objects and their categorical behaviors in this monoidal closed category. In so doing, we study various properties of the group objects in their respective categories ($\mathsf{S}\mathsf{V}$-**GCconv**,$\otimes$), and (V-**ProbCov***, $\otimes$). Furthermore, we compare these group objects with a group object in the category of convergence approach spaces, **ConvAp** attributed to Lowen, [3].

References

[1] T. M. G. Ahsanullah, David Gauld, Jawaher Al-Mufarrij, and Fawzi Al-Thukair, Enriched lattice-valued convergence groups, Fuzzy Sets and Systems **238**(2014), 71–88.
[2] H. Herrlich and D. Zhang, Categorical properties of probabilistic convergence spaces, Appl. Categor. Struct. **6**(1998), 459–513.
[3] Index Analysis, Approach theory at Work, Chapter 8, pp. 286, Springer, 2015.
[4] L. Zhang, J. Fang, and W. Wang, Monoidal closedness of L-generalized convergence spaces, Iranian J. Fuzzy Systems **16**(2019), 139–153.

Khulod Almontashery
York University

Some recent results on semi-proximal spaces

We consider the relationship between normality and semi-proximality. We prove that every normal subspace of a finite power of ω_1 is semi-proximal while the reverse implication is open. We give a consistent example of a first countable locally compact Dowker space that is not semi-proximal. We introduce a strengthening of the class of proximal spaces and semi-proximal spaces and we define the *partition game* on a set X which is equivalent to the Ulam game when the space is discrete. We show that an uncountable discrete space is proximal but not strongly proximal, and an uncountable discrete space of measurable cardinality is proximal but not even strongly semi-proximal.

Lori Alvin
Furman University

Unimodal Maps and Substitutions

We investigate unimodal maps whose kneading sequences have a nice structure. We say that a kneading sequence $\mathcal{K}(f)$ has *substitutive structure* if there exists a substitution $\theta : \mathcal{A} \to \mathcal{A}^+$ with fixed point $\mathbf{w} = \lim_{n\to\infty} \theta^n(a)$ (for some $a \in \mathcal{A}$) and a rule $\phi : \mathcal{A} \to \{0{,}1\}^+$ such that $\mathcal{K}(f) = \phi(\mathbf{w})$. We study the relationships between various dynamical properties of the unimodal map and dynamical properties of the substitution shift. In particular, we discuss conditions where $f|_{\omega(c)}$ is topologically conjugate to an odometer.

Andrea Ammerlaan
Nipissing University

Accessible Points of Chainable Continua

This talk will discuss the Nadler-Quinn problem. Posed in 1972, the problem asks if, given any chainable continuum X and any point $x \in X$, we can embed X in the plane with x accessible. In 2001, Minc constructed a particularly simple example of a chainable continuum X and point $p \in X$ for which it was not known whether p could be made accessible in a plane embedding of X. However, Anusic proved in 2018 that it was not a counterexample and that X could be embedded with p accessible. I will give an overview of this proof and briefly introduce a more recent approach to the problem.

Ana Anusic
University of Sao Paulo

Planar embeddings of chainable continua and accessibility

A *continuum* is a compact, connected, metric space, and it is *chainable* if it can be covered by an arbitrary small *chain*, or equivalently, if it can be represented as an inverse limit on intervals, where the bonding maps can be taken piecewise linear. In 1972, Nadler and Quinn asked whether, given a chainable continuum X and a point $x \in X$, there exists an embedding of X in $\mathbb{R}^2$ for which x is an accessible point. We say that a point $x \in X \subset \mathbb{R}^2$ is accessible (from the complement of X) if there is an arc $A \subset \mathbb{R}^2$ such that $A \cap X = \{x\}$. It is well-known that every chainable continuum can be embedded in the plane, but if a continuum is complicated enough, such an embedding will necessarily leave many (most) points inaccessible.

The Nadler-Quinn problem still remains open, but significant progress has been made in the recent years. In 2018, AA, H. Bruin and J. Činč gave a criterion for determining when there is an embedding of the inverse limit on intervals for which a given point is accessible, based on the notion of *zigzags*. That criterion was used by AA in 2020 to give a positive answer to the Nadler-Quinn problem for a class of chainable continua which includes an example by P. Minc (2001).

I will talk about the most recent progress on the Nadler-Quinn problem, based on my work with A. Ammerlaan and L. Hoehn. We restate and simplify the zigzag condition in terms of *radial departures* of bonding maps. Using this criterion, we give an affirmative answer to the problem of Nadler and Quinn for a large class of chainable continua.

Yunita S. Anwar
Universitas Gadjah Mada

The Direct Sum of Topological M-Injective Hulls

Let R be a topological ring and M be a topological R-module. A topological R-module E is called a topological M-injective if for every continuous monomorphism $f\colon L \to M$, where L is an open submodule of M, and for every continuous homomorphism $g\colon L \to E$, there exists a continuous homomorphism $h\colon M \to E$ that extends f. A topological M-injective hull of N is a minimal topological M-injective extension of N. An infinite direct sum of injective modules is not necessary injective. We prove that an infinite direct sum of topological M-injective modules is also topological M-injective if the direct sum is an open submodule of the direct product of topological M-injective modules. Furthermore, we prove that an infinite direct sum of topological M-injective hulls is equivalent to a topological M-injective hull of the direct sum of topological R-modules in $Top_{\sigma[M]}$ if the direct sum of topological M-injective hulls is an open submodule in the direct product of topological M-injective hulls.

Iztok Banic
University of Maribor

An inverse limit of Cantor fans

We construct a transitive mapping f on the Cantor fan F such that the inverse limit of (F, f) is homeomorphic to the Lelek fan. This is a joint work with Goran Erceg, Judy Kennedy, Chris Mouron, and Van Nall.

Adam Bartoš
Czech Academy of Sciences

Fraïssé-theoretic presentations of continua

I will give an overview of several approaches to construct and view continua as Fraïssé limits. In 2006, Irwin and Solecki gave a Fraïssé-theoretic characterization of the pseudo-arc. This was done by first constructing a pre-space of the pseudo-arc and then taking its canonical quotient. Since then many classical and some new continua have been realized and studied this way.

With Wiesław Kubiś we have developed an approximate framework for Fraïssé theory, where the pseudo-arc itself (as opposed to its pre-space) is the Fraïssé limit. In this framework we have realized the P-adic pseudo-solenoid for P any set of primes.

With Tristan Bice and Alessandro Vignati we consider another approach where we build countable posets from sequences of finite graphs and special bonding relations, and then turn the posets into compact metrizable spaces in a functorial way. The elements of the poset correspond to special basic open sets of the space. This way we keep the combinatorial nature of the building blocks while avoiding taking quotients of pre-spaces.

Dana Bartošová
University of Florida

Greatest ambit and the universal minimal flow as spaces of near ultrafilters

Our framework is that of continuous actions of topological groups on compact Hausdorff spaces called *flows*. A flow is *topologically transitive* if it has a dense orbit. Since the closure of any orbit in any flow is a subflow, topologically transitive flows are abundant in topological dynamics. For every topological group G, there is the most complicated topologically transitive flow, the greatest ambit $S(G)$. In the case of a discrete group G, $S(G)$ coincides with the Čech-Stone compactification $\beta(G)$ of all ultrafilters on G with the natural translation action. We will show Koçak and Arvasi's description $S(G)$ in terms of near ultrafilters on G and how to generalize this approach to the universal minimal flow G, that is, the most complicated minimal (with respect to inclusion) flow, and we will discuss applications.

Joseph M. Bedich
University of Pittsburgh

Using graph theory to analyze continua via finite open covers

For a continuum X, any finite open cover $\mathcal{U}$ of X defines an abstract simplicial complex. We can then analyze various graph invariants of the 1-skeleton of this simplicial complex. By considering all finite open covers and their refinements, we can lift any graph invariant to define a corresponding topological graph invariant and determine its value for the continuum X. In this talk, we specifically discuss the topological chromatic number, the topological clique number, and how these two invariants are related to covering dimension. We conclude with some discussion of results for other topological graph invariants.

Jernej Činč
University of Maribor & University of Ostrava

Topological expansion properties for Lebesgue measure-preserving circle maps

In this talk I will discuss some recent topological dynamics results in the setting of Lebesgue measure-preserving circle maps composed with rotations. In particular, I will show that our results imply there is an open dense set of Lebesgue measure-preserving circle maps satisfying a very strong expansion (locally eventually onto) property. The talk is based on the joint work with Jozef Bobok (CVUT Prague), Piotr Oprocha (AGH Krakow & University of Ostrava) and Serge Troubetzkoy (Aix Marseille).

Steven Clontz
University of South Alabama

π-Base: A semantically searchable database of topological spaces

"Topology is a dense forest of counterexamples. A usable map of the forest is a fine thing." This paraphrase from Mary Ellen Rudin's review of Steen & Seebach's Counterexamples in Topology observed the utility of having an organized way to store and discover mathematical results and references. Similar to the LMFDB or OIES, the π-Base is a community-maintained database of examples from general topology. Accepted pull requests to the π-base/data GitHub repository are automatically ingested into a Typescript application for use in a web browser at Topology.pi-Base.org. This talk will explore the sociotechnical infrastructure of this software and its possible application to other areas of mathematics.

Xabier Dominguez
Universidade da Coruña

Duality of Lipschitz-free Groups

In this talk we will consider the metric group $\mathrm{Lip}_0(X,\mathbb{T})$ of all pointed, Lipschitz functions defined on a pointed metric space X and with values on the compact group $\mathbb{T} = \mathbb{R}/\mathbb{Z}$. We will present and explore the duality between this metric group and the free Abelian group $A_d(X)$

on the space X, equipped with the Graev extension of the original metric. The analogous duality with Banach spaces instead of metric groups has remained an important area of study in Functional Analysis for several decades.

These considerations lead naturally to the topological characterization of metric dual groups, and different concepts of boundedness in both metric and topological contexts.

This is an ongoing project with M. J. Chasco and M. Tkachenko.

Othman E. Echi
King Fahd University of Petroleum and Minerals

On Skula Spaces and Quasi-homeomorphisms

Let $(X,\mathcal{T})$ be a topological space. By the *Skula topology* (or the **b**-topology) on X, we mean the topology $\mathbf{b}(\mathcal{T})$ on X with basis the collection of all $\mathcal{T}$-locally closed sets of X, the resulting space $(X, \mathbf{b}(\mathcal{T}))$ will be denoted by $\mathbf{b}(X)$. We show that the following results hold.

(1) $\mathbf{b}(X)$ is an Alexandroff space if and only if the T_0-reflection $\mathrm{T}_0(X)$ of X is a T_D-space.
(2) $\mathbf{b}(X)$ is a Noetherian space if and only if $\mathrm{T}_0(X)$ is finite.
(3) If we denote by X^* the Alexandroff extension of X, then $\mathbf{b}(X^*) = (\mathbf{b}(X))^*$ if and only if $(X,\mathcal{T})$ is a Noetherian quasisober space.

A space is said to be clopen if its open sets are also closed.Clop$(\mathcal{T})$ of $\mathcal{T}$ is clopen topology with indiscrete components the form $C_x = \overline{\{x\}} \cap \mathcal{O}(x)$, where $x \in X$ and $\mathcal{O}(x)$ is the intersection of all open sets of X containing x (equivalently, $C_x = \{y \in X : \overline{\{x\}} = \overline{\{y\}}\}$). We show that $\mathrm{Clop}(X) = \mathbf{b}\,(\mathbf{b}(X))$.

Goran Erceg
University of Split

A transitive homeomorphism on the Lelek fan

We construct a transitive homeomorphism on a Lelek fan. First, I will review our results from a previous paper where we showed that the Lelek fan is homeomorphic to an infinite Mahavier product of a single closed relation on [0,1]. Then I will present how we used them to prove that the shift map on that Mahavier product is transitive. And finally, from that we construct a transitive homeomorphism on the Lelek fan. Our homeomorphism has exactly one periodic point which itself is a fixed point and its entropy is positive. (this is joint work with Iztok Banic and Judy Kennedy)

Daniel S. Farley
Miami University

Finiteness properties of some groups of piecewise projective homeomorphisms

Lodha and Moore defined a group G of piecewise projective homeomorphisms of the line, here called the Lodha-Moore group. This group is notable for being the first known to have three properties simultaneously: 1) G is nonamenable; 2) G has no free subgroups; 3) G has type F_∞ (first proved by Lodha).

In this talk, I will give another proof that G has type F_∞ (i.e., there is a $K(G,1)$ space with finitely many cells in each dimension). I will also consider various generalizations (ten in all) that also have type F_∞. The proof uses a general framework for proving finiteness properties developed by the Bruce Hughes and the author.

Ziqin Feng
Auburn University

Topological Groups and ω^ω-bases

A property $\mathcal{P}$ is a three-space property for topological groups if for every topological group G and a closed normal subgroup H of G, both H and G/H having $\mathcal{P}$ implies that G has $\mathcal{P}$. It is a classical result that having an ω-base (i.e. first-countable, metrizable) is such a property. It is now known that having an ω^ω-base is a three-space property for topological groups. This is an open question raised by Gabriyelyan etc in 2015. In this talk, we'll present some positive results related to this open question.

Sina Greenwood
The University of Auckland

Manifolds and inverse limits of set valued functions on intervals

We discuss inverse sequences $(f_i \colon [0,1] \to 2^{[0,1]})_{i=1}^{\infty}$ of upper semi-continuous set-valued functions whose inverse limits are manifolds, with or without boundary. If for any $n > 1$, such an inverse limit is an n-manifold without boundary, then the inverse limit must be an n-torus, and hence for every $n > 1$ there is a variety of n-dimensional continua that cannot be realised as an inverse limit of set-valued functions on intervals.

Nick Gurski
Case Western Reserve University

Universal diagrams and coherence for symmetric monoidal functors etc

In practice, we try to avoid using the symmetric monoidal category axioms to verify if a diagram commutes, and instead rely on a coherence theorem that reduces this question to comparing two different elements of some symmetric group. Intuitively, the symmetric monoidal functor axioms seem to imply that the same process should work for functors, but I was unable to find a theorem stating exactly that. In this talk, I will give some examples of the kinds of diagrams I am interested in verifying, and then give a 2-monadic interpretation of the problem and its solution.

This is joint work with Niles Johnson.

Dirk Hofmann
Universidade de Aveiro

Enriched compact Hausdorff spaces

An important source of inspiration for our research over the past years has been the work [6] of Nachbin about topological spaces equipped with an additional partial order relation, subject to a compatibility condition. Our second main motivation stems from [5] where metric spaces are studied as categories enriched in the quantale $[0,\infty]$. Combining these ideas, and motivated by [7], we started to investigate *quantale-enriched compact Hausdorff spaces* which, by definition, are quantale-enriched categories equipped with a compatible compact Hausdorff topology. By design, this notion generalises Nachbin's partially ordered compact spaces; but also classical compact metric spaces can be viewed as enriched compact Hausdorff spaces. In this talk we give an overview of results describing the connection of enriched compact Hausdorff spaces with enriched topological spaces similar to the equivalence of partially ordered compact spaces and stably compact spaces [4], we provide an enriched version of the Vietoris construction [1], and discuss enriched Stone dualities [2, 3].

References

[1] D. Hofmann, *The enriched Vietoris monad on representable spaces*, Journal of Pure and Applied Algebra, 218 (2014), pp. 2274–2318.
[2] D. Hofmann and P. Nora, *Enriched Stone-type dualities*, Advances in Mathematics, 330 (2018), pp. 307–360.
[3] ——, *Duality theory for enriched Priestley spaces*, Journal of Pure and Applied Algebra, 227 (2023), p. 107231.
[4] J. Lawson, *Stably compact spaces*, Mathematical Structures in Computer Science, 21 (2011), pp. 125–169.
[5] F. W. Lawvere, *Metric spaces, generalized logic, and closed categories*, Rendiconti del Seminario Matemàtico e Fisico di Milano, 43 (1973), pp. 135–166. Republished in: Reprints in Theory and Applications of Categories, No. 1 (2002), 1–37.
[6] L. Nachbin, *Topologia e Ordem*, University of Chicago Press, 1950.
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David Holgate
University of the Western Cape

Topogenous orders on faithful functors

Given a functor $F : \mathcal{A} \to \mathcal{B}$ between categories and any $B \in \mathcal{B}$, the fibre of B is the collection $F^{-1}(B) = \{A \in \mathcal{A} \mid FA = B\}$. These fibres are naturally ordered as follows:

$$A_1 \leq A_2 \Leftrightarrow \text{there exists } f : A_1 \to A_2 \text{ in } \mathcal{A} \text{ with } Ff = 1_B.$$

While (to the non-categorist) this may seem unduly abstract, we encounter such orderings in may concrete settings. For instance the usual ordering of subgroups of a group G, the ordering of topologies on a set X and in fact any partial order can all be interpreted in this way.

In this talk we will briefly discuss properties of functors F that ensure that these orders on fibres are well behaved, and then examine stronger order relations - called topogenous orders - on fibres and how they extend topological notions of closure and interior as well as open and closed maps to many other settings.

This is joint work with Josef Šlapal and Minani Iragi (Brno University of Technology).

Alejandro Illanes
Universidad Nacional Autónoma de México

Problems on hyperspaces of continua

Given a continuum X, in this talk we will discuss open problems related with some of the hyperspaces of X, including partial answers, solutions and non-solved questions. We will focus on problems related to contractibility and Whitney levels.

Daniel Ingebretson
University of Illinois at Chicago

Hausdorff and packing measure of some digital and Luroth expansions

We compute the exact Hausdorff and packing measure of sets of digital and Luroth expansions that are missing digits beyond a given threshold.

Akira Iwasa
Howard College

Infinite antichain and forcing

Let P be a partially ordered set. We show that if P has no infinite antichain, then forcing cannot add an infinite antichain to P. We also show that if P is scattered and has no infinite antichain, then forcing cannot add a new initial segment to P.

Ivan Jelić
University of Split

The finite coarse shape groups

(joint work with Nikola Koceić-Bilan)

Given an arbitrary category C and its dense and full subcategory $D \subseteq C$, an *abstract finite coarse shape category* $Sh^{\circledast}_{(C,D)}$ is constructed using an inverse system approach. In this talk, we investigate properties concerning some recently introduced finite coarse shape invariants and the $k-th$ *finite coarse shape group* of a pointed topological space and the $k-th$ *relative finite coarse shape group* of a pointed topological pair.

Furthermore, we define the notion of *finite coarse shape group sequence* of a pointed topological pair (X, X_0, x_0) and show that, for any pointed topological pair, the corresponding finite coarse shape group sequence is a chain. On the other hand, we construct an example of a pointed pair of metric continua whose finite coarse shape group sequence fails to be exact. Finally, using the aforementioned pair of metric continua together with a pointed dyadic solenoid, we show that finite coarse shape groups, in general, differ from both shape and coarse shape groups.

John H. Johnson Jr.
Ohio State University

Affine cubes via a Khintchine-type recurrence theorem for sets with positive relative upper density

We introduce “relative” notions of upper density on the positive integers, which essentially generalize upper Banach density, and connect them with previously considered relative notions of syndetic and thick sets. Our main result is a Khintchine-type recurrence theorem for certain sets with positive relative upper density. The proof of this theorem uses a combination of two elementary intersection lemmas and the algebraic structure of the Stone-Čech compactification. As one combinatorial application we show certain sets with positive relative upper density contains affine cubes–a simple (and historically the earliest) combinatorial structure found among a large class of objects studied in Ramsey Theory. (Joint work with Florian Richter.)

Teja Kac
University of Maribor

On rigid continua and inverse limits

We give mapping theorems for certain families of rigid continua; i.e., we give a mapping theorem for stars, paths and cycles of Cook continua, and simple fans of Cook continua. We also introduce the degree of rigidity of a continuum, the notion of $\frac{1}{n}$-rigid continua and 0-rigid continua, and provide some existence theorems for $\frac{1}{n}$-rigid and 0-rigid continua. We also construct a non-trivial infinite family of pairwise non-homeomorphic continua X with the property that for any sequence (f_n) of continuous surjections $f_n : X \to X$, the inverse limit $\varprojlim\{X, f_n\}_{n=1}^{\infty}$ is homeomorphic to X. Explicitly, we show that for each positive integer n, every $\frac{1}{n}$-rigid continuum and every simple fan of Cook continua has this property.

This is joint work with Iztok Banič and Matevž Črepnjak, both from the University of Maribor.

Alexandre Karassev
Nipissing University

Discrete homogeneity

A space is called discrete homogeneous if for any two its discrete subsets there exists a homeomorphism of the space mapping one of these subsets to another. We discuss various versions of this property and its connections with other types of homogeneity. We show that a connected non-compact manifold of dimension at least 2 is strongly isotopically discrete homogeneous if and only if it has one end in the sense of Freudenthal compactification.

Lee Kennard
Syracuse University

Graph systoles and torus representations

In joint work with Michael Wiemeler (Muenster) and Burkhard Wilking (Muenster), a one-to-one correspondence was found between an important class of torus representations and combinatorial objects called regular matroids. Combinatorial arguments, computations on finite graphs, and known results for matroids then imply information on the fixed-point set data for subgroups of the torus. Our main area of interest for applications is to Riemannian geometry. For example, we prove Hopf's Euler characteristic positively conjecture for metrics invariant under a 5-dimensional torus action. Previous results of this form required lower bound on the torus rank that grew to infinity as a function of the manifold dimension.

Judy Kennedy
Lamar University

The Lelek fan as an inverse limit of Cantor fans

(This is joint work with Iztok Banic and Goran Erceg). We have constructions of transitive homeomorphisms on both the Lelek fan and the Cantor fan.

Abdul G. Khan
University of Delhi

On Minimal Expansivity and Bi-asymptotic c-Expansivity

In this talk, we will first see some examples on the unit interval admitting expansivity nature such as pointwise minimal expansivity and bi-asymptotic c-expansivity. We will explore the bi-asymptotically c-expansive maps on metric spaces and will discuss its relationship with other variants of expansivity such as bi-asymptotically expansive maps and N-expansive maps. We will finally establish the spectral decomposition theorem for bi-asymptotically c-expansive continuous surjective maps with the shadowing property on compact metric spaces. This talk is based on joint-work with Rohit Nageshwar and Prof. Tarun Das.

Jimmie Lawson
Louisiana State University

Old and New in Non-T_2

Assuming a level of separation in topological spaces that minimally includes Hausdorff separation has been widely prevalent in the study of general topology. However, over the course of the last 60 years there has been a widening stream of research treating the topology of not necessarily Hausdorff and non- Hausdorff spaces, so that a considerable and reasonably coherent body of material has emerged. In this talk we attempt to sketch some major contours of this theory. One major theme arises from the insight that non-Hausdorff topology may be viewed as a confluence of general topology and order theory via the order of specialization of the topology.

This aspect often leads to an interesting and fruitful interplay of both topological and order-theoretic methods that leads beyond the boundaries of what can be obtained by restricting to topological or order-theoretic approaches alone.

We next consider some of the basic topological properties and classes of spaces that have arisen in the study of T_0-spaces. These include sober spaces, well-filtered spaces, and monotone convergence spaces (also called d-spaces). We consider stud- ies of universal completions (some recent) with these (and related) properties.

A second important aspect of the theory of non-Hausdorff spaces is the fact that such topologies frequently come as one of a pair. As a key example one first notes that the theory of metric spaces generalizes to quasimetric spaces by dropping the symmetry axiom. Such spaces have a related quasimetric topology defined from the quasimetric, but also a "dual" topology defined from the quasimetric $d*(x,y) = d(y,x)$. One can then define the "patch topology," which is the join of the two topologies. An enriched theory of polish spaces has emerged in recent years by moving from metrics to quasimetrics. Compact Hausdorff spaces generalize in the T_0-setting to stably compact spaces, which have in many ways a richer, fuller theory. Each stably compact space has a dual stably compact space, what is sometimes called DeGroot duality. One can also start with a partially ordered topological space and consider the "dual" topologies of open upper resp. lower sets. An old open problem about complete regularity in this context was recently solved

A final theme is the close connections that have existed between the "domain theory" pioneered by Dana Scott and co-workers and related developments in non-Hausdorff topology. "Domains" are special partially ordered sets that have been used for modeling denotational semantics and other constructs in theoretical computer science, but the theory has developed hand-in-glove with a variety of aspects of the theory of T_0-spaces. A crucial insight is that adding an appropriate T_0-topology to an ordered structure can allow one to develop the theory of the ordered structure in a much more comprehensive way.

Christopher Leininger
Rice University

Billiards, geometry, and symbolic coding

Given a polygon in the Euclidean or hyperbolic plane, its billiard flow on the tangent bundle has trajectories that describe the paths of particles in the polygon (the billiard trajectories) traveling along straight lines and "bouncing" off the sides. Labeling the sides of the polygon the billiard flow determines a symbolic coding we call the bounce spectrum, which is the set of biinfinite sequences of labels corresponding to the sides encountered by all trajectories. A natural question asks the extent to which the bounce spectrum determines the shape of the polygon. For both Euclidean and hyperbolic polygons, there are nontrivial constructions of polygons with the same bounce spectrum that are not isometric/similar. In this talk, I'll describe these constructions, and then results of joint work with Duchin, Erlandsson, and Sadanand stating that these are in fact the only ways in which non-isometric/non-similar polygons can have the same bounce spectrum.

Michelle C. LeMasurier
Hamilton College

Bratteli Diagrams for Bounded Topological Speedups

A bounded topological speedup of a Cantor minimal system is a minimal system (X, S), where $S(x) = T^{p(x)}(x)$ for some bounded function $p : X \to \mathbb{Z}^+$, or any system topologically conjugate to such an (X, S). Assuming the system (X, T) is represented by a properly ordered Bratteli diagram $\mathcal{B}$, we provide a method for constructing a new, perfectly ordered Bratteli diagram $\widetilde{\mathcal{B}}$ that represents the sped-up system (X, S). The diagram $\widetilde{\mathcal{B}}$ relates back to $\mathcal{B}$ in a manner that enables us to see how certain dynamical properties are preserved under speedup. As an application, in the case that (X, T) is a substitution minimal system, we show how to use $\widetilde{\mathcal{B}}$ to write an explicit substitution rule that generates the sped-up system (X, S), answering an open question from Alvin, Ash, Ormes 2018.

Rory Lucyshyn-Wright*
Brandon University

Algebra with spaces or structures as arities: Algebraic structures in enriched categories

This talk will be an introduction to recent joint work with Jason Parker in which we extend prior work on enriched monads and Lawvere theories to provide a convenient framework for algebraic structures whose arities can be arbitrary objects of categories $\mathcal{C}$-enriched over base categories $\mathcal{V}$ that need not be locally presentable, including various categories of topological structures. Lawvere's elegant approach to (essentially syntactic) presentations of Lawvere theories [1,II.2] was generalized to the setting of finitary monads on a locally finitely presentable category $\mathcal{C}$-enriched in a locally finitely presentable base $\mathcal{V}$ by Kelly and Power [2], so that the arities of the operations are finitely presentable objects of $\mathcal{C}$. Bourke and Garner [3] generalized such presentations of monads to a setting where the arities are allowed to be objects of an arbitrary small *subcategory of arities* $\mathcal{J} \hookrightarrow \mathcal{C}$, i.e. a small full subcategory that is dense in the enriched sense, while still requiring that $\mathcal{V}$ and $\mathcal{C}$ be locally presentable (which entails that the objects of $\mathcal{J}$ are again α-presentable for a fixed α). In recent work [4,5,6], and forthcoming further extensions, we not only dispense with the assumption of local presentability by defining an axiomatic setting for enriched Lawvere theories and monads for general subcategories of arities, but also we introduce a formalism of *parametrized operations* and *diagrammatic presentations* that facilitates the definition of enriched algebraic structures.

For simplicity, in this talk we shall focus on the case where we begin with a *locally bounded closed category* $\mathcal{V}$, noting that every topological category over Set carries at least one locally bounded closed structure. Given a subcategory of arities $\mathcal{J} \hookrightarrow \mathcal{C}$ in a $\mathcal{V}$-enriched category $\mathcal{C}$, such as $\mathcal{V}$ itself, we consider algebraic structures in $\mathcal{C}$ consisting of an object A of $\mathcal{C}$ (the *carrier*) equipped with *parametrized $\mathcal{J}$-ary operations*; the latter are morphisms $f : \mathcal{C}(J,A) \to \mathcal{C}(C,A)$ in

$\mathcal{V}$, or equivalently $f : \mathcal{C}(J,A) \otimes C \to A$ in $\mathcal{C}$ for the enriched copower $\otimes$, where J and C are objects of $\mathcal{J}$ and of $\mathcal{C}$, respectively, called the *arity* and *parameter*. For example, if $\mathcal{C} = \mathcal{V} = \mathrm{CGTop}$ is the category of compactly generated spaces, then A, J, C are compactly generated spaces, and such operations include not only continuous finitary operations and various continuous actions, but also homotopies between continuous finitary operations as well as continuous families of operations on spaces of paths, etcetera. We provide a formalism for defining *varieties* of $\mathcal{J}$-ary algebraic structures of the above kind, each equipped with a small family of parametrized $\mathcal{J}$-ary operations satisfying a small family of diagrammatic equations. Assuming that $\mathcal{J}$ is small and that $\mathcal{C}$ is $\mathcal{V}$-*sketchable* (as is the case when $\mathcal{C} = \mathcal{V}$ and for various categories of structures in $\mathcal{V}$), we show that each such variety of $\mathcal{J}$-ary algebraic structures in $\mathcal{C}$ is a $\mathcal{V}$-enriched category that is monadic over $\mathcal{C}$ by way of an enriched monad that is $\mathcal{J}$-*nervous* in a sense adopted from Bourke and Garner [3]. We show that varieties of $\mathcal{J}$-ary algebraic structures in $\mathcal{C}$ are equivalently $\mathcal{J}$-nervous monadic $\mathcal{V}$-categories over $\mathcal{C}$, and we develop a novel intrinsic characterization of these. We establish an equivalence between $\mathcal{J}$-nervous monads and $\mathcal{J}$-*theories* and a dual equivalence between $\mathcal{J}$-theories and $\mathcal{J}$-ary varieties, and we develop a Lawverian structure-semantics adjunction for $\mathcal{J}$-theories, among various further results.

* Joint work with Jason Parker. We acknowledge the support of the Natural Sciences and Engineering Research Council of Canada. Nous remercions le Conseil de recherches en sciences naturelles et en génie du Canada de son soutien.

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Sergio Macias
Universidad Nacional Autónoma de México

Local Homogeneity and the Uniform Property of Effros

Kathryn F. Porter wrote a nice paper about several definitions of local homogeneity (Local homogeneity, JP Journal of Geometry and Topology, 9 (2009), 129-136). In this paper, she mentions that G. S. Ungar defined a uniformly locally homogeneous space (Local homogeneity, Duke Math. J., 34 (1967), 693-700). We realized that this notion is very similar to what we call the uniform property of Effros (On Jones' set function $\mathcal{T}$ and the property of Kelley for Hausdorff continua, Topology Appl., 226 (2017), 51-65). Here, we compare the uniform property of Effros with the uniform local homogeneity. We also consider other definitions of local homogeneity given in Porter's paper and compare them with the uniform property of Effros.

Atish J. Mitra
Montana Technological University

Euler Characteristic Surfaces as topological summaries of dynamical systems

Euler Characteristic Surfaces were introduced by the author and his collaborators (Phys. Fluids 32, 123310 (2020)) as as a multi-scale spatiotemporal map of an evolving flow situation. In this talk we will discuss further work in this direction. We discuss connections of the Euler Characteristic Surface with various proximity complexes and some stability results. We demonstrate a metric that identifies the stability regime of a given flow pattern, besides distinguishing between different flow systems. We demonstrate the efficacy of our approach on both simulated and experimental systems.

Ulises Morales-Fuentes
CINC, Universidad Autónoma del Estado de Morelos

Rectangles inscribed in plane compact sets, plane continua and plane dense sets

A plane set X admits an inscribed polygon P, if every vertex of a polygon similar to P lies in X. In 1977 H. Vaughan proved that every homeomorphic copy of S^1 in $\mathbb{R}^2$ admits at least one inscribed rectangle. In this talk we present some examples of disconnected one dimensional compact sets such that every homeomorphic copy of them in $\mathbb{R}^2$ admits at least one inscribed rectangle. Also we show that a dense union of disjoint arcs always admits at least one inscribed rectangle, provided that one of the arcs is a line segment. Finally, we use the non-embeddability in $\mathbb{R}^3$ of the cone of certain graphs to classify locally connected plane continua that inscribe rectangles.

Frederic Mynard
New Jersey City University

On pointfree convergence and pointfree approach theory

Goubault-Larrecq and Mynard introduced a framework for pointfree convergence [1] that allows to represent convergence approach spaces as pointfree convergence spaces [2]. This representation, however, lacks some structure to easily characterize AP within the pointfree convergence spaces corresponding to CAP. A full-fledge pointfree index theory requires new ingredients. I will present progress in this direction, which is joint work with Emilio Angulo-Perkins.

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Van Nall
University of Richmond

Positive entropy of shift maps on inverse limits with set valued functions

This is joint work with Iztok Banic, Judy Kennedy, and Rene Gril Regina. We introduce a structure called a (k, ϵ)-return that is sufficient for positive entropy in a shift map on an inverse limit with a closed relation on a compact set. We also show that this structure is necessary in the case that the the relation is finite.

Peter J. Nyikos
University of South Carolina

Normality in thin-tall spaces and the cardinal $\mathfrak{b}$

Definition 1. A *thin-tall space* is one that has an uncountable set of levels indexed by the countable ordinals; each level is countably infinite and discrete in the relative topology, and each neighborhood of a point on level α meets all levels indexed by ordinals $< \alpha$.

The following theorem has a strange status: the further properties of the known examples depend on the status of the small uncountable cardinal $\mathfrak{b}$. The ones when $\mathfrak{b} = \omega_1$ are very different from the ones when $\mathfrak{b} > \omega_1$.

Theorem 1. There exists a normal, locally compact, thin-tall space.

When $\mathfrak{b} = \omega_1$, the example can be perfectly normal; but there are models of $\mathfrak{b} > \omega_1$ where they cannot even be hereditarily normal. On the other hand, when $\mathfrak{b} > \omega_1$, the construction can be done in ZFC, and one only needs $\mathfrak{b} > \omega_1$ to show normality.

Definition 2. A *sub-Ostaszewski space* is a locally compact, locally countable, uncountable Hausdorff space in which every open subset is either countable or co-countable. An *Ostaszewski space* is a countably compact sub-Ostaszewski space.

Every sub-Ostaszewski space is hereditarily separable and thin-tall. Every Ostaszewski space is perfectly normal, but the following problem is open:

Problem 1. If there is a sub-Ostaszewski space, must there be one that is normal?

It is easy to show that if $\mathfrak{b} > \omega_1$, then every sub-Ostaszewski space is normal. But it is also routine to construct a non-normal sub-Ostaszewsi space when $\mathfrak{b} = \omega_1$ using $\clubsuit$.

Problem 2. Is $\clubsuit$ enough to imply the existence of a normal sub-Ostaszewski space?

Piotr Oprocha
AGH University of Krakow

On entropy of map induced on the hyperspace of continua in dimension one

Any map on compact metric space X induces in a natural way a map on the hyperspace of all compact subsets of X. When the space X is additionally connected, it makes sense to consider dynamics on (sub)hyperspace of all continua. In this talk we will present relations between entropy of base map and the map induced on the hyperspace, with main emphasis on one-dimensional spaces as the base space X.

Jason Parker
Brandon University

Free algebras of topologically enriched multi-sorted algebraic theories

Classically, *multi-sorted algebraic theories* and their free algebras have been fundamental in mathematics and computer science. Given a set $\mathcal{S}$ of *sorts*, a *(classical) $\mathcal{S}$-sorted signature* is a set Σ of *operation* (or *function*) *symbols* equipped with an assignment to each $\sigma \in \Sigma$ of a finite tuple $(S_1, \ldots, S_n)$ of *input sorts* and an *output sort* $S \in \mathcal{S}$, in which case we write $\sigma : S_1 \times \ldots \times S_n \to S$. A *$\Sigma$-algebra* $\mathbb{A}$ is an $\mathcal{S}$-indexed family of *carrier sets* $\mathcal{A} = (A_S)_{S \in \mathcal{S}}$ equipped with, for each $\sigma : S_1 \times \ldots \times S_n \to S$ in Σ, a function $\sigma^{\mathbb{A}} : A_{S_1} \times \ldots \times A_{S_n} \to A_S$. A *(classical) $\mathcal{S}$-sorted algebraic theory* is a pair $\mathcal{T} = (\Sigma, \mathcal{E})$ consisting of a classical $\mathcal{S}$-sorted signature Σ and a set $\mathcal{E}$ of *equations* between *Σ-terms*, and a *$\mathcal{T}$-algebra* is a Σ-algebra that satisfies these equations. Writing $\mathcal{T}\text{-}\mathsf{Alg}$ for the category of $\mathcal{T}$-algebras, the forgetful functor

$U^{\mathcal{T}} : \mathcal{T}\text{-}\mathsf{Alg} \to \mathsf{Set}^{\mathcal{S}}$ that sends a $\mathcal{T}$-algebra to its underlying $\mathcal{S}$-indexed family of carrier sets is well known to have a left adjoint $F^{\mathcal{T}} : \mathsf{Set}^{\mathcal{S}} \to \mathcal{T}\text{-}\mathsf{Alg}$.

The purpose of the present work is to generalize the foregoing from the classical (Set-*enriched*) context to the context of enrichment in a symmetric monoidal category $\mathcal{V} = (\mathcal{V}, \otimes, I)$ that is equipped with a *topological (forgetful) functor* $|-| : \mathcal{V} \to \mathsf{Set}$ that is strict monoidal (with respect to the cartesian monoidal structure on Set). Prominent examples of such $\mathcal{V}$ include:

- Various categories of topological spaces (which were previously considered for the present purposes in [2]), as well as the category of measurable spaces.

- The categories of models of *relational Horn theories without equality*, including the category of preordered sets and monotone functions, and the category of (extended) pseudo-metric spaces and non-increasing functions.

- The categories of *quasispaces* (a.k.a. *concrete sheaves*) on *concrete sites*, which include the categories of diffeological spaces, quasi-Borel spaces [5], bornological sets, (abstract) simplicial complexes, and convergence spaces [1, 3].

Given such a category $\mathcal{V}$ and a set $\mathcal{S}$ of sorts, we define a notion of $\mathcal{V}$-*enriched* $\mathcal{S}$-*sorted signature* Σ, which extends the notion of classical $\mathcal{S}$-sorted signature by requiring that each operation symbol $\sigma \in \Sigma$ be equipped also with a *parameter object* P in $\mathcal{V}$. A Σ-*algebra* $\mathbb{A}$ is then an $\mathcal{S}$-indexed family $\mathcal{A} = (A_S)_{S \in \mathcal{S}}$ of *carrier objects of* $\mathcal{V}$ equipped with, for each $\sigma : S_1 \times \ldots \times S_n \to S$ in Σ with parameter object P, a $\mathcal{V}$-*morphism* $\sigma^{\mathbb{A}} : P \otimes (A_{S_1} \times \ldots \times A_{S_n}) \to A_S$. A $\mathcal{V}$-*enriched* $\mathcal{S}$-*sorted algebraic theory* $\mathcal{T}$ is a pair $\mathcal{T} = (\Sigma, \mathcal{E})$ consisting of a $\mathcal{V}$-enriched $\mathcal{S}$-sorted signature Σ and a set $\mathcal{E}$ of Σ-*equations* (generalizing the classical equations), while a $\mathcal{T}$-*algebra* is a Σ-algebra that satisfies these equations. We write $\mathcal{T}\text{-}\mathsf{Alg}$ for the category of $\mathcal{T}$-algebras, which is equipped with a forgetful functor $U^{\mathcal{T}} : \mathcal{T}\text{-}\mathsf{Alg} \to \mathcal{V}^{\mathcal{S}}$ that sends a $\mathcal{T}$-algebra to its underlying $\mathcal{S}$-indexed family of carrier objects of $\mathcal{V}$.

We show that every $\mathcal{V}$-enriched $\mathcal{S}$-sorted algebraic theory $\mathcal{T} = (\Sigma, \mathcal{E})$ has an underlying classical $\mathcal{S}$-sorted algebraic theory $|\mathcal{T}| = (|\Sigma|, |\mathcal{E}|)$. The forgetful functor $U^{\mathcal{T}}$ has a left adjoint $F^{\mathcal{T}} : \mathcal{V}^{\mathcal{S}} \to \mathcal{T}\text{-}\mathsf{Alg}$, and we show that the resulting adjunction $F^{\mathcal{T}} \dashv U^{\mathcal{T}} : \mathcal{T}\text{-}\mathsf{Alg} \to \mathcal{V}^{\mathcal{S}}$ is a *(strict) lifting* of the adjunction $F^{|\mathcal{T}|} \dashv U^{|\mathcal{T}|} : |\mathcal{T}|\text{-}\mathsf{Alg} \to \mathsf{Set}^{\mathcal{S}}$. We use this result to establish concrete descriptions of free $\mathcal{T}$-algebras, which have an even more explicit and (*countably*) *inductive* character when $\mathcal{V}$ is cartesian closed. We provide several examples of $\mathcal{V}$-enriched multi-sorted algebraic theories, and we discuss the close connection between $\mathcal{V}$-enriched multi-sorted algebraic theories and the (presentations of) $\mathcal{V}$-enriched monads studied in [6, 7, 8], as well as with the *relational (single-sorted) algebraic theories* of [4] and the *quantitative (single-sorted) algebraic theories* of [9], which are special (single-sorted) instances of the theories considered here.

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Artur Piekosz
Cracow University of Technology

Stone and Esakia Dualities in tame topology

I will give an introduction on how tame topology has evolved from Grothendieck sites and show variants of Stone Duality and Esakia Duality for small and locally small spaces. I will need some theory of (up-)spectral spaces and a useful notion of a special bornology in a bounded distributive lattice. Examples related to o-minimality will be given.

Daniel A. Ramras
Indiana University-Purdue University Indianapolis

Quillen fibers, homotopy colimits, and Lurie's Higher Seifert-Van Kampen Theorem

Quillen's Fiber Theorem (Theorem A) allows one to recognize weak equivalences between (classifying spaces of) categories and has become an essential tool in algebraic K-theory and topological combinatorics. I'll explain a new perspective on Quillen's theorem in terms of the Grothendieck construction. Consequences of this viewpoint include k-connected versions of both Quillen's theorem (generalizing work of Björner for posets) and the Bousfield-Kan homotopy cofinality theorem for homotopy colimits. As an application, we extend and generalize Lurie's Higher Seifert-Van Kampen Theorem. This result allows one to recover a space as the homotopy colimit of an open cover, subject to a sort of homotopical fineness condition. We prove a k-connected version that applies to both open covers (with some point-set restrictions) and covers of CW-complexes by subcomplexes.

Tom Richmond
Western Kentucky University

Hybrid topologies on the real line

The Hattori topology $H(A)$ on $\mathbb{R}$ is a hybrid of the Euclidean and lower-limit topologies, with points of A having Euclidean neighborhood bases and all other points having lower-limit neighborhood bases. We investigate other hybrid topologies using combinations of the Euclidean, lower-limit, upper-limit, left-ray, right-ray, and discrete topologies. In particular, we consider conditions for quasi-metrizability for some of these hybrids.

Stephen E. Rodabaugh
Youngstown State University

A Survey and Exposition of Sub-Hausdorff Separation Axioms

J. T. Denniston (Kent State University), A. Melton (Kent State University),
S. E. Rodabaugh* (Youngstown State University), J. K. Tartir (Youngstown State University)

*presenter

In memoriam: Prof. S. Floyd Barger

Abstract. This talk samples the authors' recently submitted work of the same title, which logically orders a large number (23) of sub-Hausdorff separation axioms (known and new), and thereby augments known relationships amongst T_0, S_0 (quasi-sobriety), S_1 (sobriety), T_1, and T_2.

This work emphasizes hereditary S_0, hereditary S_1, deleted S_0, deleted S_1, (locally) strong S_0, (locally) strong S_1, and T_{sc} (singleton closure property). Properties of the Kolmogorov functor $K : \mathbf{Top} \to \mathbf{Top}_0$ are developed and applied. Examples, often based on modified Fort spaces and Šierpinski space, show non-reversibility of strategic implications and correct published errors. Some theorems are given both point-set and spectrum proofs to provide additional insights.

Results include: locally Hausdorff strictly implies T_1 + hereditary S_1; multiple characterizations of hereditary S_0 and hereditary S_1, filling gaps in Barger (1997); local characterizations of S_0 and S_1; a new spectrum-based proof of Hong's (1975) theorem that K both preserves and reflects S_0, as well as extensions of Hong's theorem to each of hereditary S_0, LSS_0, T_sc, and the conjunction of hereditary LSS_0 and T_sc; a characterization of T_1 in terms of T_0 and T_sc; and in the presence of T_sc, LSS_0 is hereditary and hence implies hereditary S_0, adding to Gierz, *et al* (2003).

Rene G. Rogina
University of Maribor

The two lines inverse limit and end-point-generated fans

For positive numbers $r < 1$ and $\rho > 1$, let $L_{r,\rho}$ be union of two line segments in [0,1] $\times$ [0,1], one from (0,0) to (1,r) and the other from (0,0) to ($\frac{1}{\rho}$,1). It was proven for all such r and ρ that, if r and ρ never connect, then the Mahavier product of $L_{r,\rho}$'s is homeomorphic to the Lelek fan. We show that for all such r and ρ, if r and ρ do connect, the Mahavier product of $L_{r,\rho}$'s is the union of a countable family of Cantor fans with additional properties regarding the limits of sequences of end-points. We further define what it means for a fan to be end-point-generated.

Vinod P. Saxena
Jiwaji University

Finite Element Analysis and Applications to Human Physiological Heat Regulation

Finite Element Analysis is an important interdisciplinary topic of core and applied mathematics. It's mathematical study involves functional analysis and topology as associated domains are fractured in regular or irregular parts (elements).

The applications of finite element analysis and method are tremendous for solving problems in physics, engineering and biology. In particular, Finite Element approach has been widely used in continuum problems. This gives flexible multi-parameter based situations, a technique to reach close solutions. This approach is supported by numerical and computational operations.

In this lecture application of above in heat regulation problems in in-vivo tissues of human body peripheral regions are discussed. These are formulated in terms of partial differential equations incorporating significant parameters like micro-circulation rate, metabolic heat generation rate and other physical quantities which may vary with position and time. The variation patterns are different in different elements.

The governing equations are transformed into variational integrals which are redefined in each subregion due to different biological conditions. Approximate solutions are generated in terms classical functions. All these local solutions are assembled to arrive at global solution in which unknown quantities are estimated. Finally, numerical values are computed for different situations of the human body.

Josef Šlapal
Brno University of Technology

α-sequential closure operators for digital topology

For every ordinal $\alpha > 0$, an α-sequential closure operator is defined in such a way that the closure of a set A is determined by certain α-indexed sequences formed by points of A. The α-sequential closure operators are studied and it is shown that the connectivity with respect to them is a certain type of path connectivity. This makes it possible to apply α-sequential closure operators in solving problems based on employing a convenient connectivity such as problems of digital image processing. One such application is presented showing that α-sequential closure operators may be used for defining digital Jordan curves.

Ignat Soroko
University of North Texas

Property R_∞ for Artin groups

A group G has property R_∞ if for every automorphism ϕ of G the number of twisted ϕ-conjugacy classes is infinite. This property is motivated by the topological fixed point theory, and has been a subject of active research. Among the groups which have this property are hyperbolic and relatively hyperbolic groups, mapping class groups, generalized Baumslag-Solitar groups and some others. However, the general picture of which groups have this property is quite elusive. In a joint project with Matthieu Calvez, we establish property R_∞ for some spherical and affine Artin groups by utilizing their close relation to certain mapping class groups of punctured surfaces.

Isar Stubbe
Université du Littoral-Côte d'Opale

A logical analysis of Banach's fixpoint theorem

Banach's fixpoint theorem [1] from 1922 says that every contraction on a non-empty complete metric space admits a unique fixpoint. The gist of the proof is wonderfully simple: take any element x of the space (X,d) and, iterating the contraction $f\colon X \to X$, prove that the sequence $(f^n x)_n$ is Cauchy. In the complete space (X,d) this sequence converges, and one then shows that it does so to a (necessarily unique) fixpoint of f.

In 1972, Lawvere [6] showed that metric spaces are a particular instance of enriched categories, and explained how convergence of Cauchy sequences is then an instance of representability of left adjoint distributors. He strongly advocated the point of view that "specializing the constructions and theorems of general category theory we can deduce a large part of general metric space theory." It is thus natural to investigate whether fixpoint theorems still make sense in the vast context of enriched categories: this is precisely the subject of this talk (and of our paper [2]).

More precisely, we fix a quantale (= a posetal cocomplete monoidal closed category) Q, and work with categories, functors and distributors enriched in Q. Our contribution shows that fixpoint theorems for contractions on Q-categories depend on the interplay between three essential parameters. Indeed, a given contraction must be "strong enough" (we shall measure its strength by means of a control function); the category on which it acts must be "complete enough" for the Picard iteration to converge to a fixpoint (we shall take this to be Cauchy-completeness in the sense of Lawvere); but we also need sufficiently strong algebraic properties of the underlying quantale Q to allow for the formulation of precisely that convergence.

In concreto, we shall prove a fixpoint theorem for Cauchy-complete Q-categories that holds whenever the quantale Q has an underlying continuous lattice and the contraction is controlled by a sequentially lower-semicontinuous function on Q. As examples we find the classical Banach fixpoint theorem for metric spaces, and Boyd and Wong's [3] generalization thereof (taking the underlying quantale to be the positive real numbers); but we also formulate new results for fuzzy ordered sets (when working over a left-continuous t-norm [5]) and for probabilistic metric spaces (now the quantale is the tensor product of the positive reals with a left-continuous t-norm [4]).

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Perfectly meager sets in the transitive sense and the Hurewicz property

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We work in the Cantor space with the usual group operation $+$. A set X is *perfectly meager in the transitive sense* if for any perfect set P there is an F_σ-set F containing X such that for every point t the intersection $F \cap (t+P)$ is meager in the relative topology of $t+P$. A set X is *Hurewicz* if for any sequence $\mathcal{U}_0, \mathcal{U}_1, \ldots$ of open covers of X, there are finite families $\mathcal{F}_0 \subseteq \mathcal{U}_0, \mathcal{F}_1 \subseteq \mathcal{U}_1, \ldots$ such that the family $\{\bigcup \mathcal{F}_n : n \in \omega\}$ is a γ-cover of X, i.e., the sets $\{n : x \notin \bigcup \mathcal{F}_n\}$ are finite for all points $x \in X$. Nowik proved that each Hurewicz set which cannot be mapped continuously onto the Cantor set is perfectly meager in the transitive sense. We present results related to the question, whether the same assertion holds for each Hurewicz set with no copy of the Cantor set inside.

AMS classification: 54D20, 03E17

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Smallness in Topology

Quillen's notion of small object and the Gabriel-Ulmer notion of finitely presentable or generated object are fundamental in homotopy theory and categorical algebra. Do these notions always lead to rather uninteresting classes of objects in categories of topological spaces, such as all finite discrete spaces, or just the empty space, as the examples and remarks in the existing literature may suggest?

This article demonstrates that the establishment of full characterizations of these notions (and some natural variations thereof) in many familiar categories of spaces can be quite challenging and may lead to unexpected surprises. In fact, we show that there are significant differences in this regard even amongst the categories defined by the standard separation axioms, with the T_1-separation condition standing out. The findings about these specific categories lead us to insights also when considering rather arbitrary full reflective subcategories of the category of all topological spaces.

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Disks with holes

It is well known that a disk with a finite number n of holes in it has fundamental group isomorphic to a free group on n generators. The situation is more complicated when the disk has infinitely many holes. We look at two situations of particular interest.

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Topological fractals

A compact metric space X is called a metric fractal if there are finitely many contractions of X into itself whose images cover X. A compact metrizable space X is called a topological fractal if there are finitely many continuous selfmaps of X whose images cover X and the images of all finite compositions of these maps form a null sequence. A 1985 conjecture by M. Hata is claiming that every Peano continuum is a topological fractal. Recently it was proved that every Peano continuum with a free arc is a topological fractal. Jointly with K. Karasová we proved that every Peano continuum with uncountably many local cut-points is a topological fractal as well.

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Turing Machine Simulation and Isoperimetric Functions of Groups

An S-machine is a computational model which resembles a multi-tape, non-deterministic Turing machine that works over group words rather than the "positive words" comprising the corresponding free monoid. This model has proved fruitful since its introduction in the late 1990s, playing a key role in the solutions to several long-standing open problems in group theory. We will discuss a recent development in the theory, producing fundamental S-machines with improved computational complexity bounds. Finally, we conclude by considering some potential implications of this improvement, including a fundamental link between the decidability of the word problem and the Dehn function.

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Hemicompactness, P-spaces and related topics in the absence of the Axiom of Choice

Several results from the following article will be shown: K. Keremedis, A. R. Olfati and E. Wajch, "*On P-spaces and G_δ-sets in the absence of the Axiom of Choice*," to appear in the Bull. Belgian Math. Soc.-Simon Stevin, vol. 30 (2) (2023) (about 39 pages). Furthermore, some of the newest theorems (due to E. Wajch), concerning hemicompactness and k_ω-spaces in in Zermelo-Fraenkel set theory without the Axiom of Choice, will be presented. For example, it will be shown that the principle of countable multiple choices (CMC) is equivalent to each of the following statements: (1) every σ-compact, locally compact Hausdorff space is paracompact; (2) for every hemicompact space X, it holds that X is locally compact at every point at which X is first-countable. The statement "every countable direct sum of hemicompact spaces is a

σ-compact space" is independent of ZF. The Principle of Weak Dependent Choices implies the following: (3) every topological space X which admits a k_ω-decomposition consisting of compact Hausdorff subspaces of X is a normal Hausdorff space; (4) every finite product of Hausdorff k_ω-spaces is a k_ω-space.

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Connections between real-enriched categories and real-valued topological spaces

While generalized metric spaces are categories enriched over the quantale ([0,∞], $\geq$, + ,0), approach spaces introduced by R. Lowen in 1989 are topological spaces valued in ([0,∞], $\geq$,+,0). When we view ([0,∞], $\geq$, + ,0) as the table of truth-values of a generalized logic, generalized metric spaces and approach spaces are then many-valued ordered sets and many-valued topological spaces, respectively. The theories of generalized metric spaces and approach spaces have been extended to the context that the table of truth-values is a commutative and unital quantale, resulting in quantale-enriched categories and quantale-valued topological spaces. Connections between toplogical spaces, quantale-enriched categories and quantale-valued topological spaces are the core object of Quantitative Domain Theory. This talk presents a brief introduction to connections between quantale-enriched categories and quantale-valued topological spaces in the case that the quantale is the unit interval together with a continuous t-norm.